Learning latent functions for causal discovery

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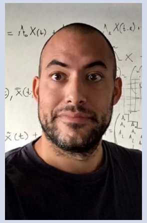

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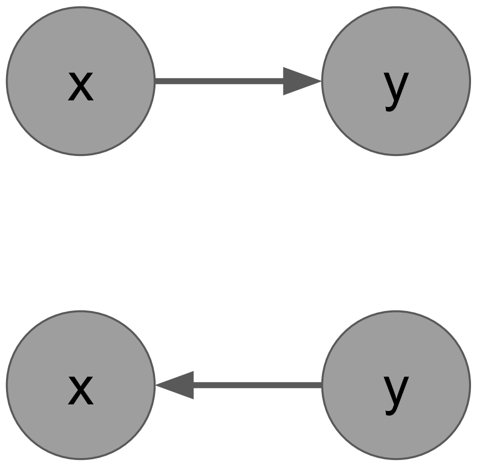
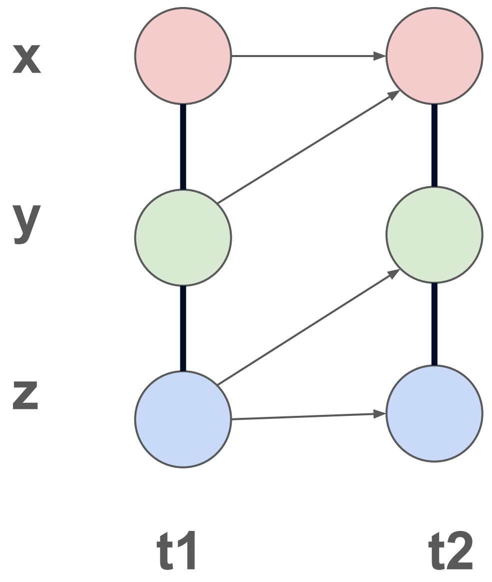
Bivariate causal discovery algorithm for non–additive data

A method to "plug-in" after the Markov equivalence class has been estimated

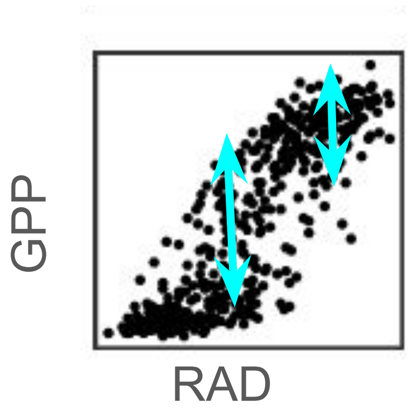




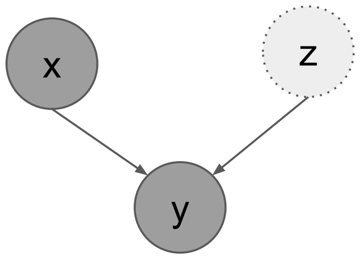
Causal insufficiency widespread in Earth system sciences

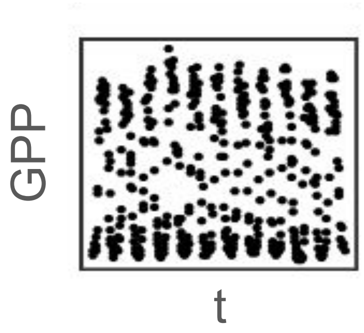
Non-additive data important for Earth system science

1. weak form of non causal sufficiency



2. can generate structured data e.g. spatial, temporal



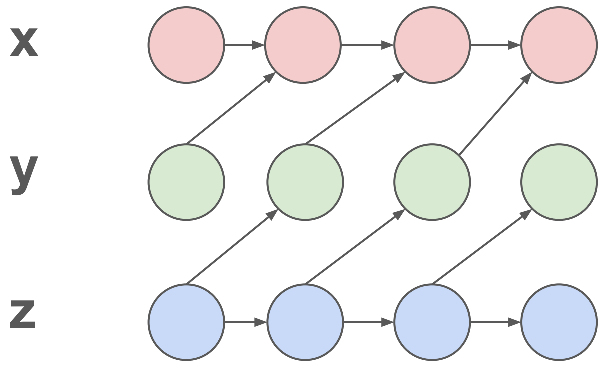


Instantaneous relationships widespread in Earth system sciences

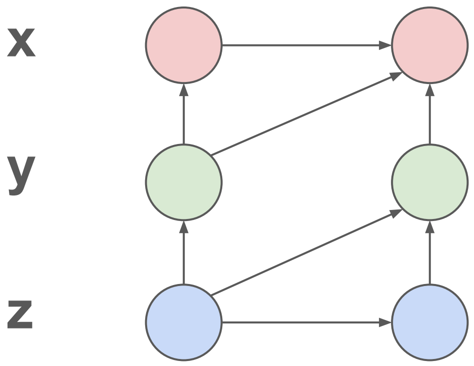
- “Instantaneous” relationships often occur in practice due to systems observed at lower resolution than the fundamental mechanisms.

- In this case additional assumptions necessary to identify the causal structure.

native resolution



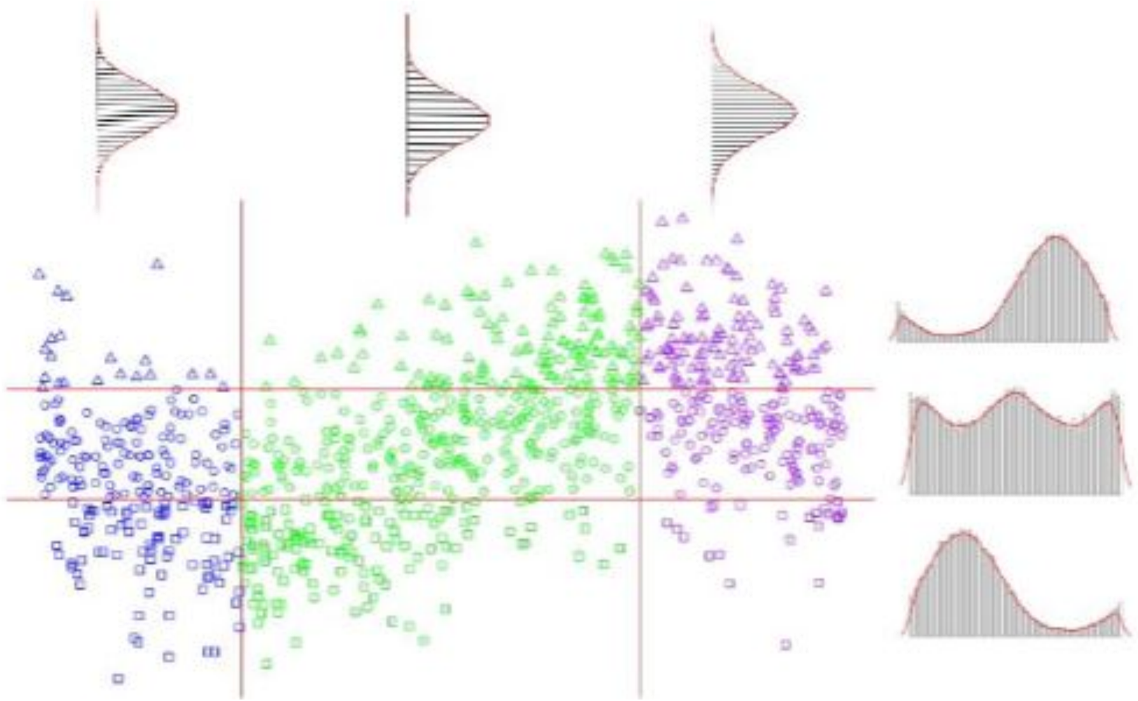
aggregated low resolution



Examples

- Daily ERA5 data
- Weekly satellite data
- harmonizationoof different products to lower temporal resolution

Independence of cause and mechanism (ICM) (Daniusis et al, 2010)



To identify causal structure when instantaneous relationships exist we need extra assumption.

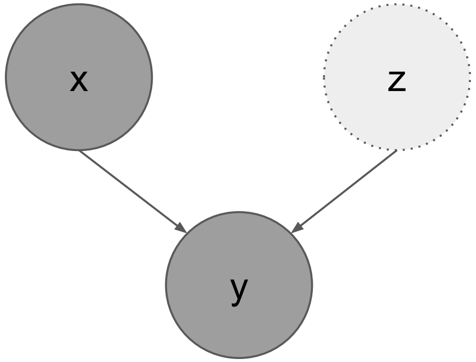
- modularity assumption

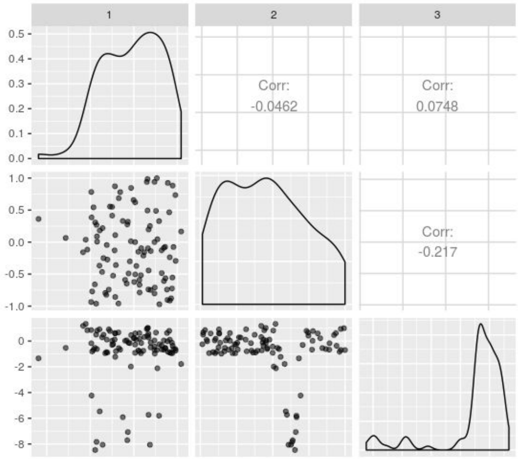
- ie no info about $p(y|x)$ in $p(x)$

Taking a step back: modeling the inducing FCM

$$y := f(x, z)$$

Latent Noise approach





conditional pdf approach

An extended ICM assumption

For a data generating mechanism $y = f(x, z)$ we make the following assumptions, following (Stegle, et al 2010)

1. Deterministic process
2. Exogenous noise z
3. Gaussian noise z
4. Algorithmic independence

Loss function for finding z penalizes assumption violations

$$L(\mathcal{Z}) = \boxed{\ln\left(nHSIC(\mathcal{X}_a, \mathcal{R}_{x\to y})\right) + \zeta \ln\left(MSE(\mathcal{R}_{x\to y})\right)}$$
$$+\eta \ln\left(nHSIC(\mathcal{X}, \mathcal{Z})\right) + \nu \ln\left(SMMD^2_{\mathcal{N}}(\mathcal{Z})\right)$$

where:

$$\mathcal{X}_a := \{(x_i, z_i)\}_{i=1}^n$$
$$\mathcal{R}_{x\to y} := \{y_i - f(x_i, z_i)\}_{i=1}^n$$
$$\hat{y} = f(x, z) = \sum_{i=1}^n \alpha_i k\big((x, z), (x_i, z_i)\big) \in \mathcal{H}_{xz}$$
$$\boldsymbol{\alpha} = (K_{xz} + n\lambda I)^{-1}\mathbf{y}$$

Deterministic process

Loss function for finding z penalizes assumption violations

$$L(\mathcal{Z}) = \ln\left(nHSIC(\mathcal{X}_a, \mathcal{R}_{x\to y})\right) + \zeta \ln\left(MSE(\mathcal{R}_{x\to y})\right)$$

$$+\eta \boxed{\ln\left(nHSIC(\mathcal{X}, \mathcal{Z})\right)} + \nu \ln\left(SMMD^2_{\mathcal{N}}(\mathcal{Z})\right)$$

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Exogenous noise

Loss function for finding z penalizes assumption violations

$$L(\mathcal{Z}) = \ln\left(nHSIC(\mathcal{X}_a, \mathcal{R}_{x\to y})\right) + \zeta \ln\left(MSE(\mathcal{R}_{x\to y})\right)$$
$$+\eta \ln\left(nHSIC(\mathcal{X}, \mathcal{Z})\right) + \nu \boxed{\ln\left(SMMD^2_{\mathcal{N}}(\mathcal{Z})\right)}$$

Gaussian noise

where:

$$\mathcal{X}_a := \{(x_i, z_i)\}_{i=1}^n$$
$$\mathcal{R}_{x\to y} := \{y_i - f(x_i, z_i)\}_{i=1}^n$$
$$\hat{y} = f(x, z) = \sum_{i=1}^n \alpha_i k\big((x, z), (x_i, z_i)\big) \in \mathcal{H}_{xz}$$
$$\boldsymbol{\alpha} = (K_{xz} + n\lambda I)^{-1}\mathbf{y}$$

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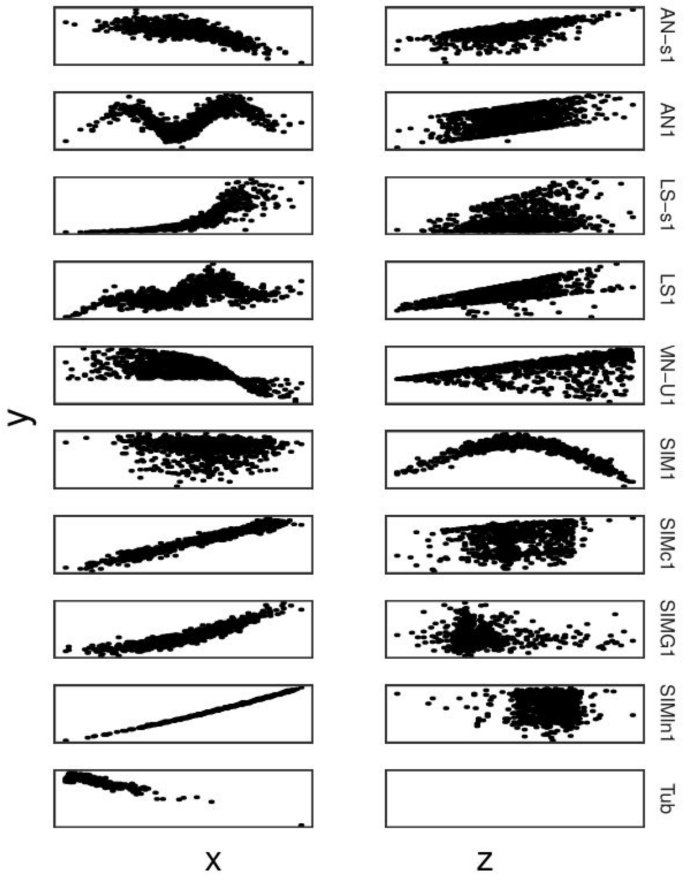
Enforce soft assumptions

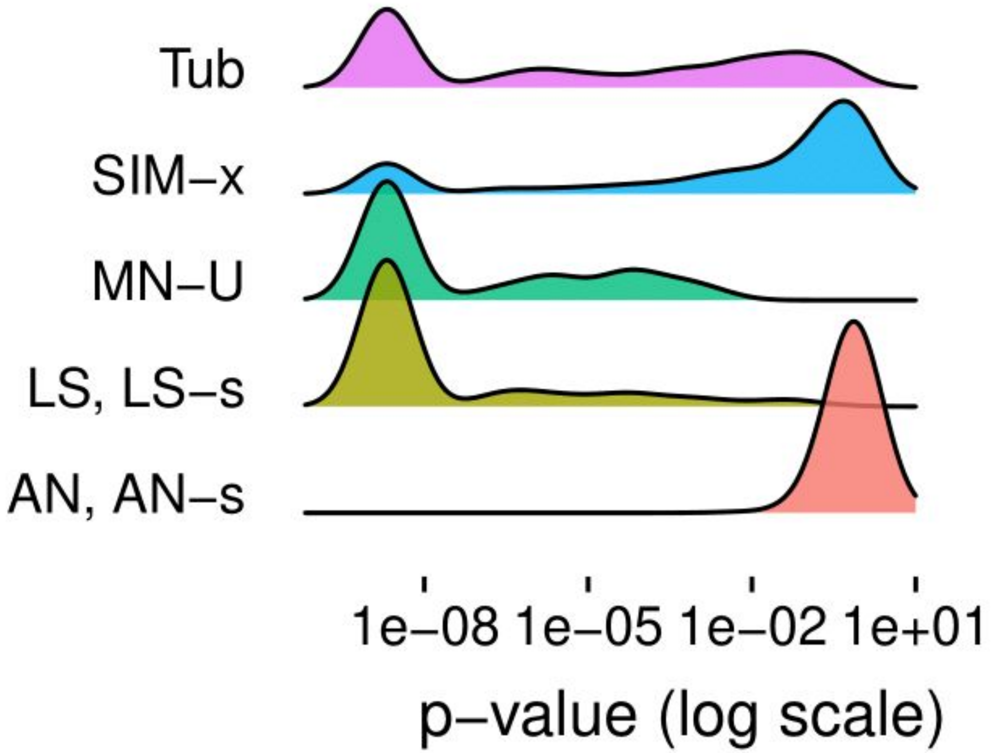
Advantages

- Rank relative importance of assumptions

- Relaxing determinism assumption: use non-additvity as causal signal:
  - model misspecification (anti causal direction)
  - estimation error (both directions)
  - asymmetry assumption: model misspecification generates more non-additivity

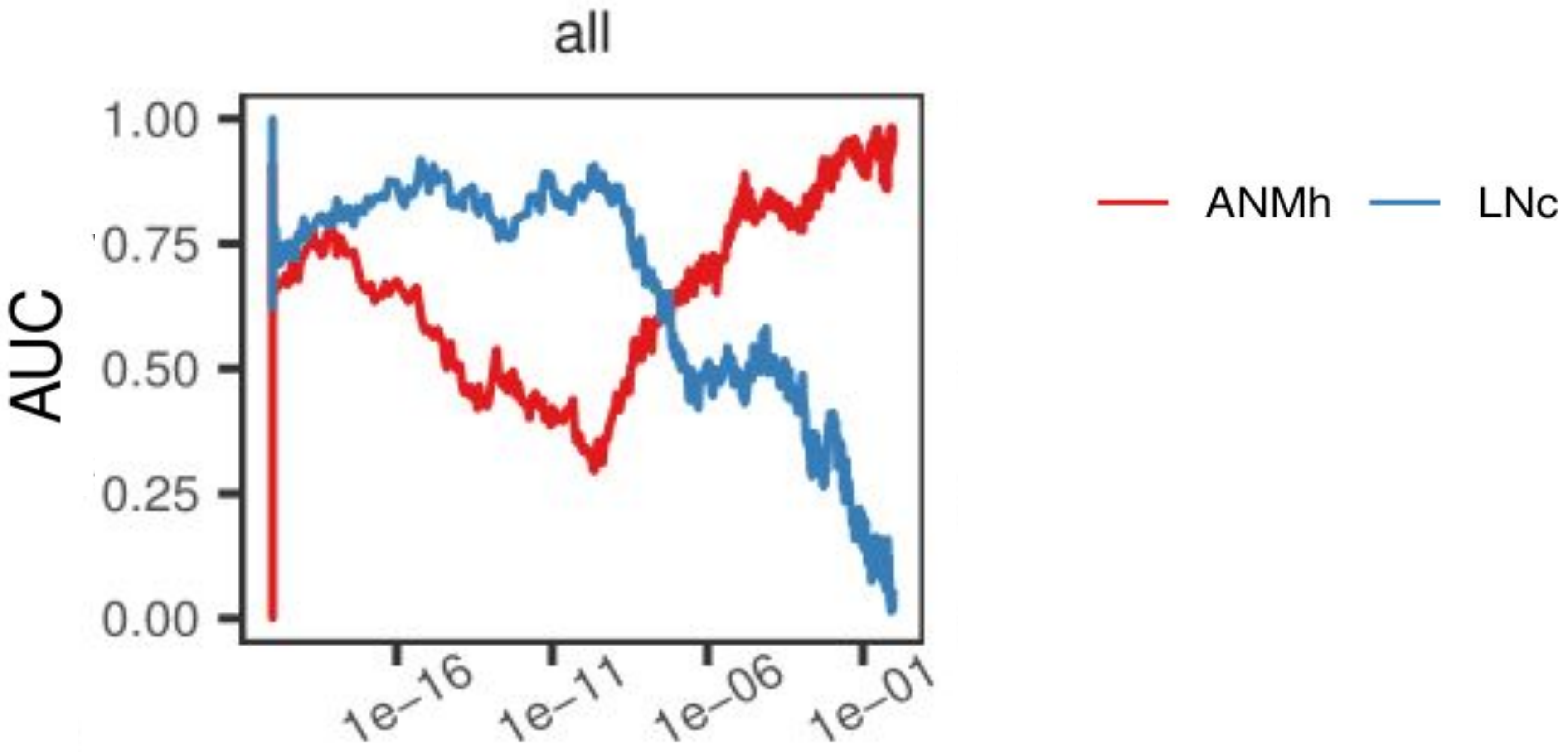
IID data & additivity hypothesis test



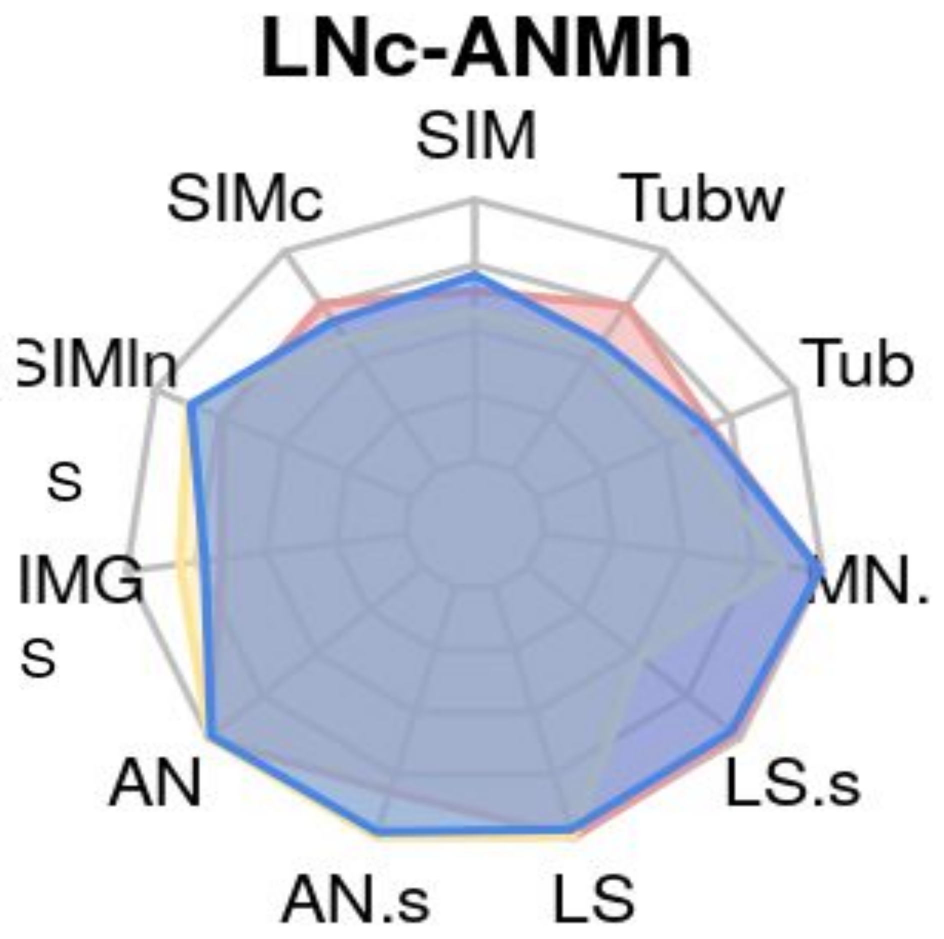


$$S = \min\{nHSIC(\mathcal{X}, \mathcal{R}_{x\to y}), nHSIC(\mathcal{Y}, \mathcal{R}_{y\to x})\}$$

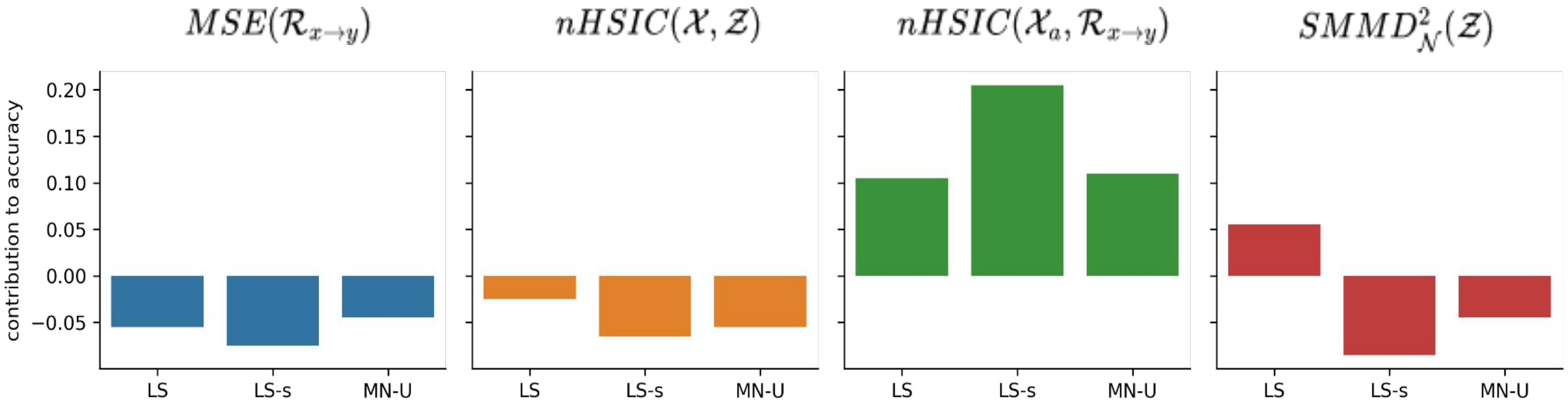
LNc method's accuracy improves with non-additivity



Combining LNc and ANMh obtains SOTA performance

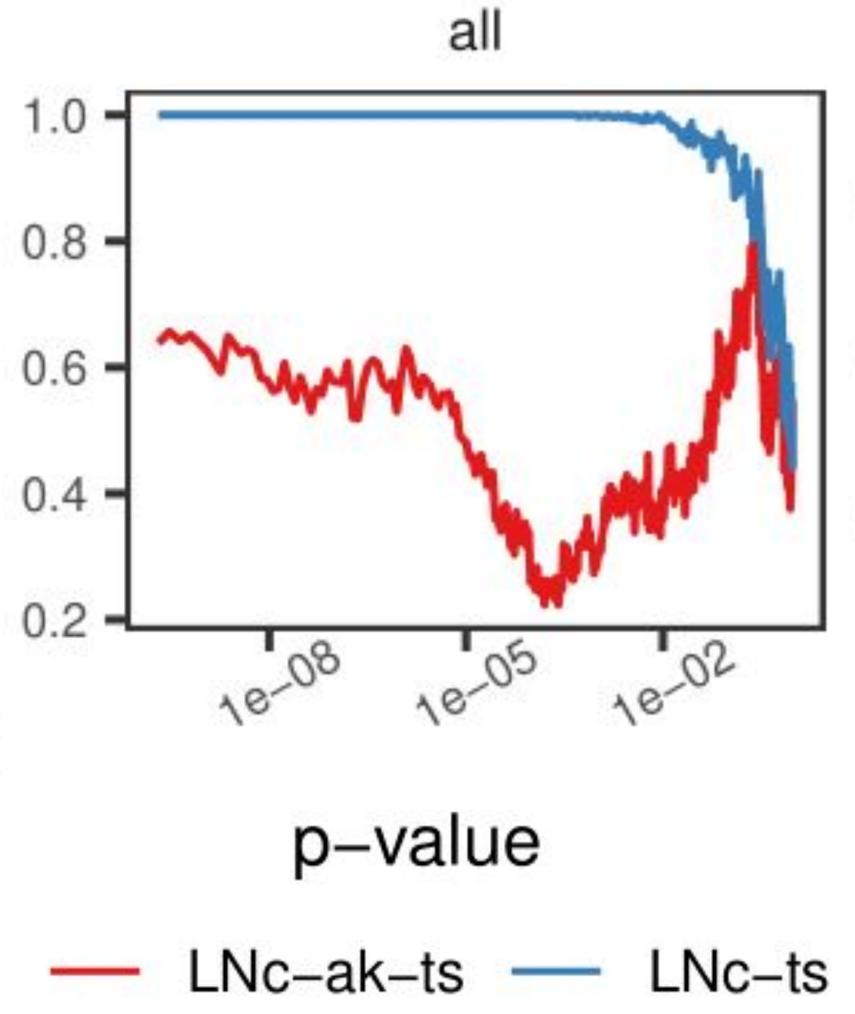


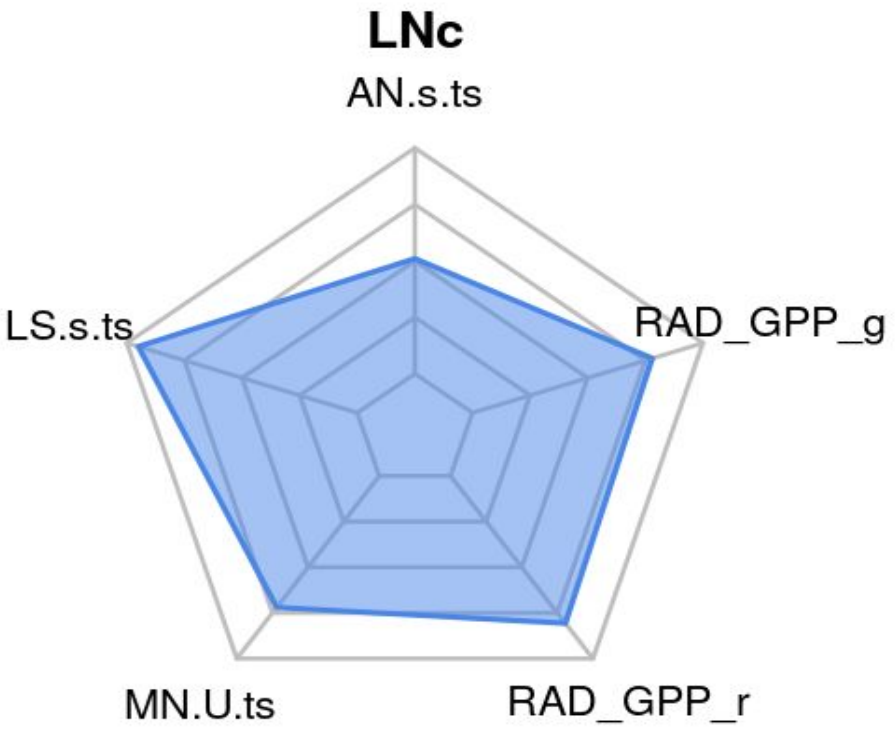
Relative importance of assumptions



- Additive residual assumption only one that needs to be implemented strictly.

LNc method's accuracy improves with non-additivity





Conclusions & contributions

- Generative approach is advantageous for extending to spatio-temporal data.
- Generative LNc method including extension to time series and an additivity hypothesis test.

Limitations

- Time series extension only applies when there is no self-dependence in effect variable
- Only suitable for cases where confounding is ruled out

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Opportunities

- When causal discovery is applied to time series at an aggregated scale instantaneous effects very common: asymmetry methods suitable.

- A soft/weak form of non-causal sufficiency can be responsible for non-additive data: need asymmetry methods suited to this setting. Non causal sufficiency very common in Earth system science.

- Can be used to complement other causal discovery algorithms that don’t make ICM assumption and can’t fully identify causal structure when “instantaneous” relationships occur.

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